The axial symmetry at finite temperature in Lattice QCD with Overlap fermions

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New Horizons for Lattice Computations with Chiral Fermions
RIKEN BNL Research center workshop – May 14, 2012

Summary:

- ✔ Motivation
- ✔ Chiral phase transition at finite temperature and axial symmetry
- ✔ Simulating dynamical (overlap) fermions
- ✔ Topology fixing and friends
- ✔ Methodology and results (quenched and $N_f=2$)
- ✔ Discussion and conclusions

People involved in this project

JLQCD group: S. Hashimoto, S. Aoki, T. Kaneko, H. Matsufuru, J. Noaki, E. Shintani

See for example POS(Lattice2010)174 (arXiv:1011.0257), PoS(Lattice 2011)188 (arXiv:1204.4519), article in prep.

Motivation

Pattern of chiral symmetry breaking at low temperature QCD

$$SU(N_f)_V \times SU(N_f)_A \times U(1)_V \times \boxed{U(1)_A} \to SU(N_f)_V \times U(1)_V$$

Well known facts about axial symmetry

- Axial anomaly – Non vanishing topological susceptibility
- Mass splitting of the η' (958 Mev) with respect to the lighter Goldsone bosons

What is the fate of the axial $U_A(1)$ symmetry
at finite temperature($T \gtrsim T_c$)?

Dirac Overlap operator, retaining the maximal amount of chiral symmetry on the lattice is, theoretically, the best way to answer this question.

●Experimental data: PHENIX(BNL) Phys.Rev. C83 (2011) 054903

"*the mass of the eta' meson is reduced by Delta-m > 200 MeV,*
at the 99.9% confidence level, in the considered model class"

Goal of the current project

Check restoration of axial $U_A(1)$ symmetry by measuring (spatial) meson correlators at finite temperature in full QCD with overlap

Degeneracy of correlators is the signal that we are looking for

$$\begin{tikzcd} \sigma(1_4 \otimes 1_2) \arrow[rr, leftrightarrow, "Chiral\ sym."] \arrow[d, leftrightarrow, "U(1)_A"'] & & \pi(i\gamma_5 \otimes \tau^a) \arrow[d, leftrightarrow, "U(1)_A"] \\ \eta(i\gamma_5 \otimes 1_2) \arrow[rr, leftrightarrow, "Chiral\ sym."] & & \delta(1_4 \otimes \tau^a) \end{tikzcd}$$

As I will show in this talk,
there is one issue to solve before attacking the real problem...

Simulating dynamical overlap fermions

In order to avoid expensive tricks to handle the zero modes of the Hermitian Wilson operator JLQCD simulations use (JLQCD 2006):

- Iwasaki action (suppresses Wilson operator near zero modes)
- Extra Wilson fermions and twisted mass ghosts to rule out the zero modes

Topology is thus fixed throughout the HMC trajectory.

The effect of fixing topology is expected to be a Finite Size Effect (actually $O(1/V)$), next slides

Fixing topology: how to deal with it at T=0 (I)

Partition function at fixed topology

$$Z_Q = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\theta\, \exp(-VF(\theta)) \qquad F(\theta) \equiv E(\theta) - i\theta Q/V$$

where the energy can be expanded $E(\theta) = \sum_{n=1}^{\infty} \frac{c_{2n}}{(2n)!}\theta^{2n} = \frac{\chi_t}{2}\theta^2 + O(\theta^4)$

Using saddle point expansion around $\theta_c = i\frac{Q}{\chi_t V}(1 + O(\delta^2))$

one obtains the Gaussian distribution

$$Z_Q = \frac{1}{\sqrt{2\pi\chi_t V}} \exp\left[-\frac{Q^2}{2\chi_t V}\right]\left[1 - \frac{c_4}{8V\chi_t^2} + O\left(\frac{1}{V^2}, \delta^2\right)\right].$$

Fixing topology: how to deal with it at T=0 (II)

From the previous partition function we can extract the relation between correlators at fixed θ and correlators at fixed Q

In particular for the topological susceptibility and using the Axial Ward Identity we obtain a relation involving fermionic quantities:

$$\lim_{|x|\to\text{large}} \langle mP(x)mP(0)\rangle_Q^{\text{disc}} \equiv \frac{1}{V}\left(\frac{Q^2}{V} - \chi_t - \frac{c_4}{2\chi_t V}\right) + O(e^{-m_\pi |x|})$$

P(x) is the flavor singlet pseudo scalar density operator

Aoki *et al*. PRD76,054508 (2007)

What is the effect of fixing Q at finite temperature?

Results

- ✔ Simulation details
- ✔ Eigenvalues distribution
- ✔ Finite temperature quenched SU(3) at fixed topology
- ✔ Meson correlators in two flavors QCD

BG/L
Hitachi SR16K

Simulation details

Pure gauge (16^3x6, 24^3x6):

Iwasaki action + top. fixing term

β	a(fm)	T (MeV)	T/Tc
2.35	0.132	249.1	0.86
2.40	0.123	268.1	0.93
2.43	0.117	280.9	0.97
2.44	0.115	285.7	0.992
2.445	0.114	288	1.0
2.45	0.1133	290.2	1.01
2.46	0.111	295.1	1.02
2.48	0.107	305.6	1.06
2.50	0.104	316.2	1.10
2.55	0.094	347.6	1.20

Two flavors QCD (16^3x8)

Iwasaki + Overlap + top. Fix

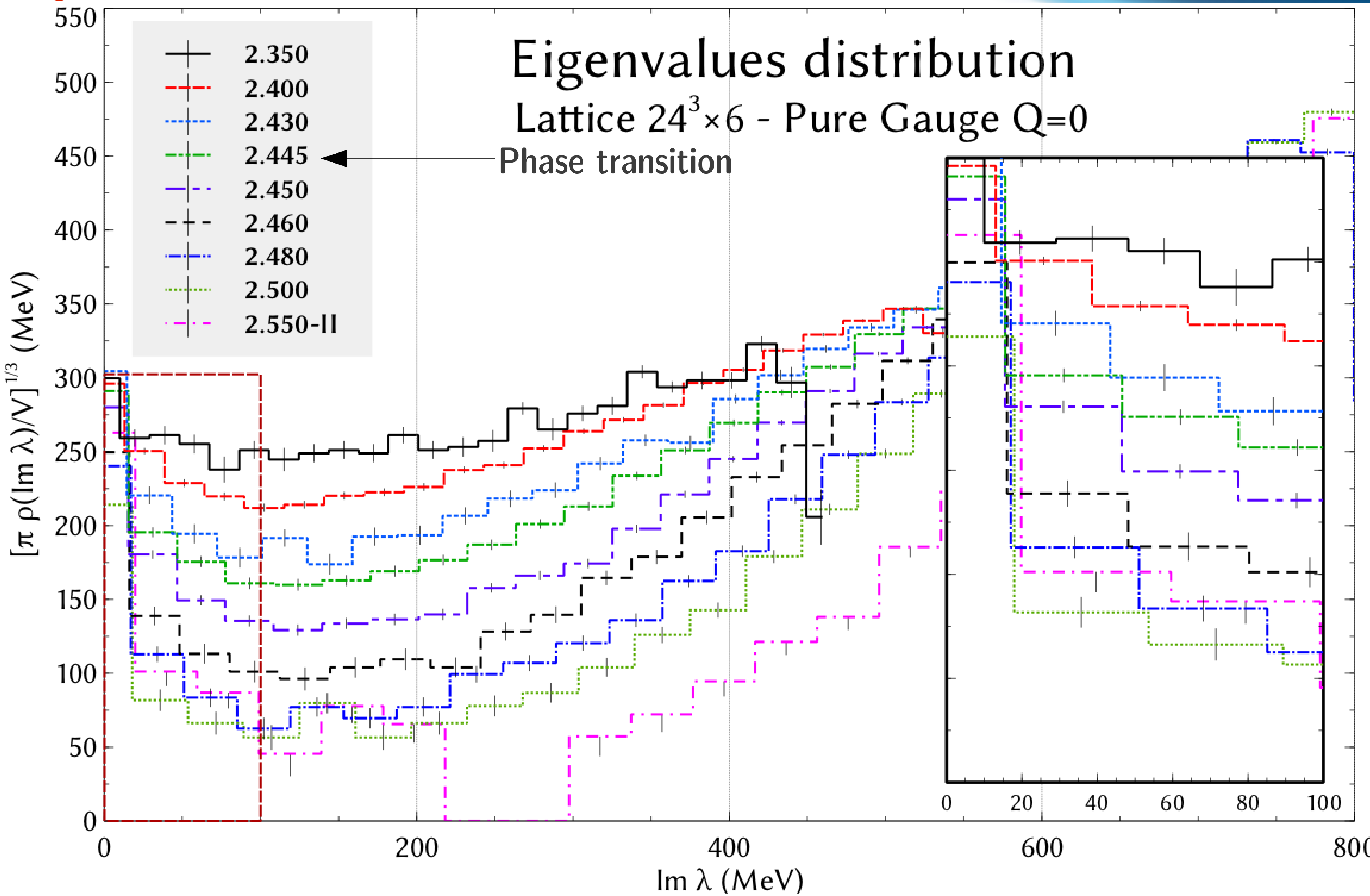
O(300) trajectories per T

am=0.05, 0.025, 0.01

β	a(fm)	T (MeV)	T/Tc
2.18	0.1438	171.5	0.95
2.20	0.1391	177.3	0.985
2.25	0.12818	192.2	1.06
2.30	0.1183	208.5	1.15
2.40	0.1013	243.5	1.35
2.45	0.0940	262.4	1.45

Axial symmetry at finite temp. with overlap fermions

Eigenvalue distribution

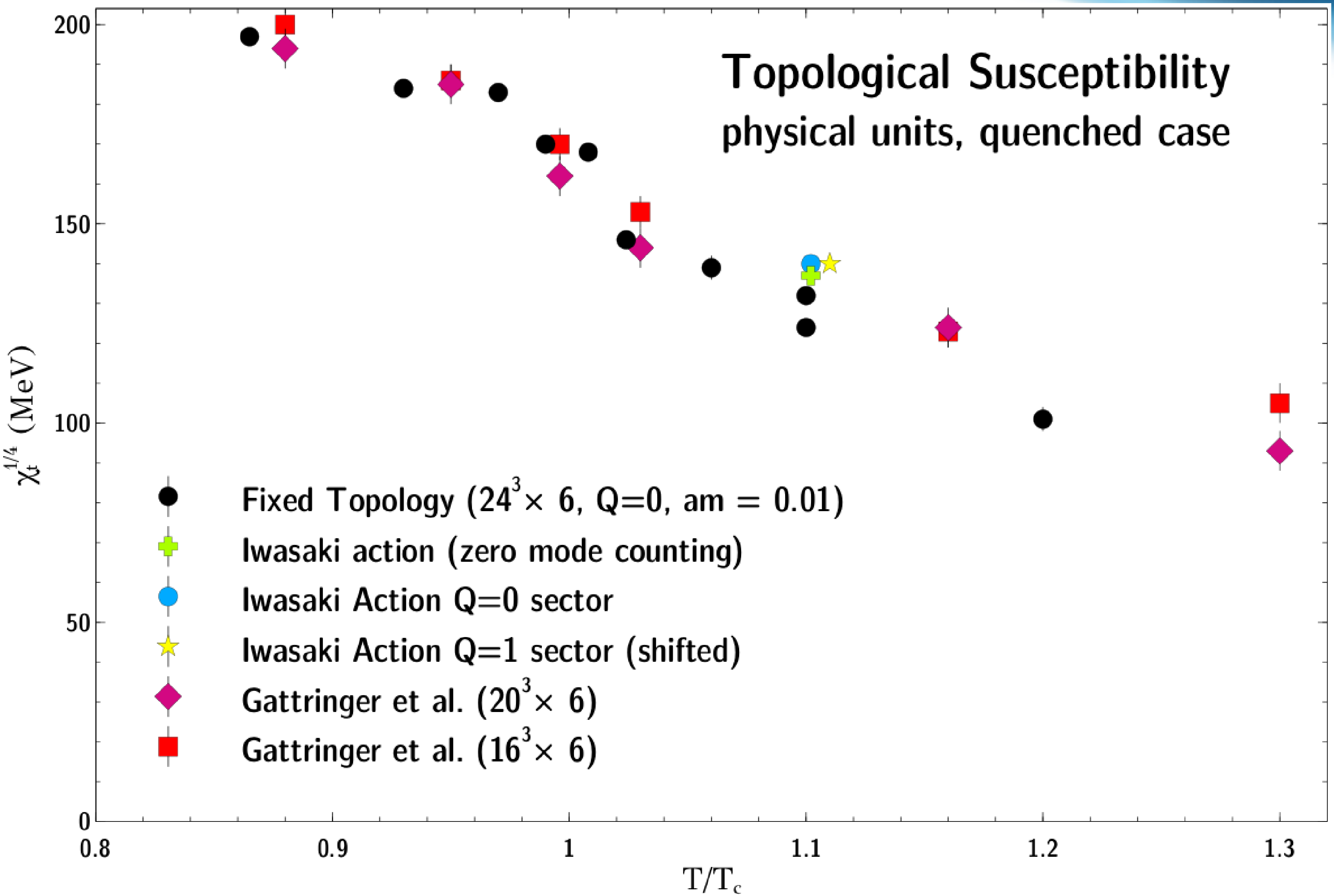


Topological susceptibility in pure gauge theory - I

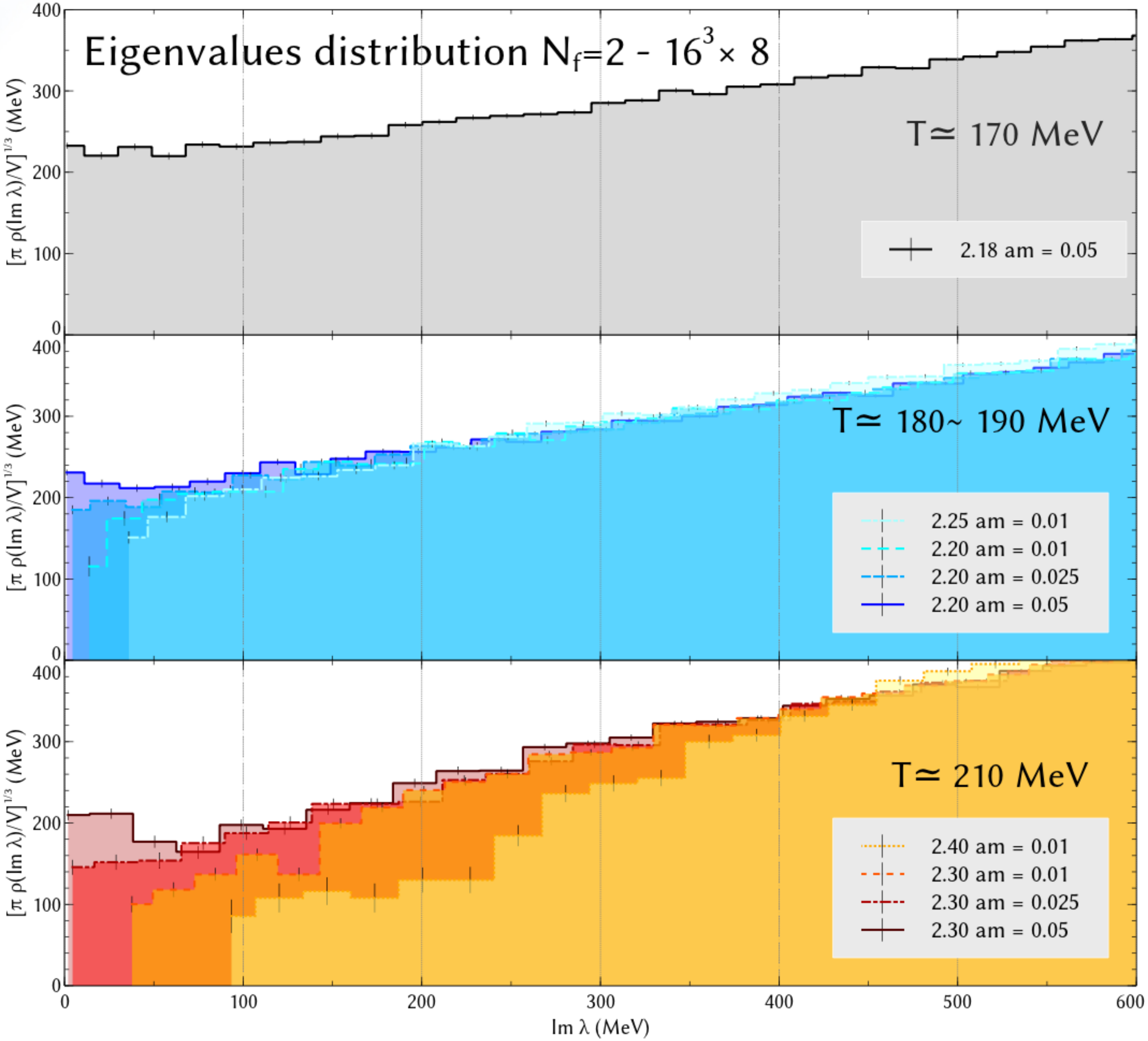
$$\lim_{|x|\to\text{large}} \langle mP(x)mP(0)\rangle_Q^{\text{disc}} \equiv \frac{1}{V}\left(\frac{Q^2}{V} - \chi_t - \frac{c_4}{2\chi_t V}\right) + O(e^{-m_\pi |x|})$$

- (Spatial) Correlators are always approximated by the first 50 eigenvalues
- Pure gauge: double pole formula for disconnected diagram
- Topological susceptibility estimated by a joint fit of connected and disconnected contribution.

Topological susceptibility in pure gauge theory - II



Full QCD – Eigenvalues



Effect of axial symmetry on the Dirac sperctrum

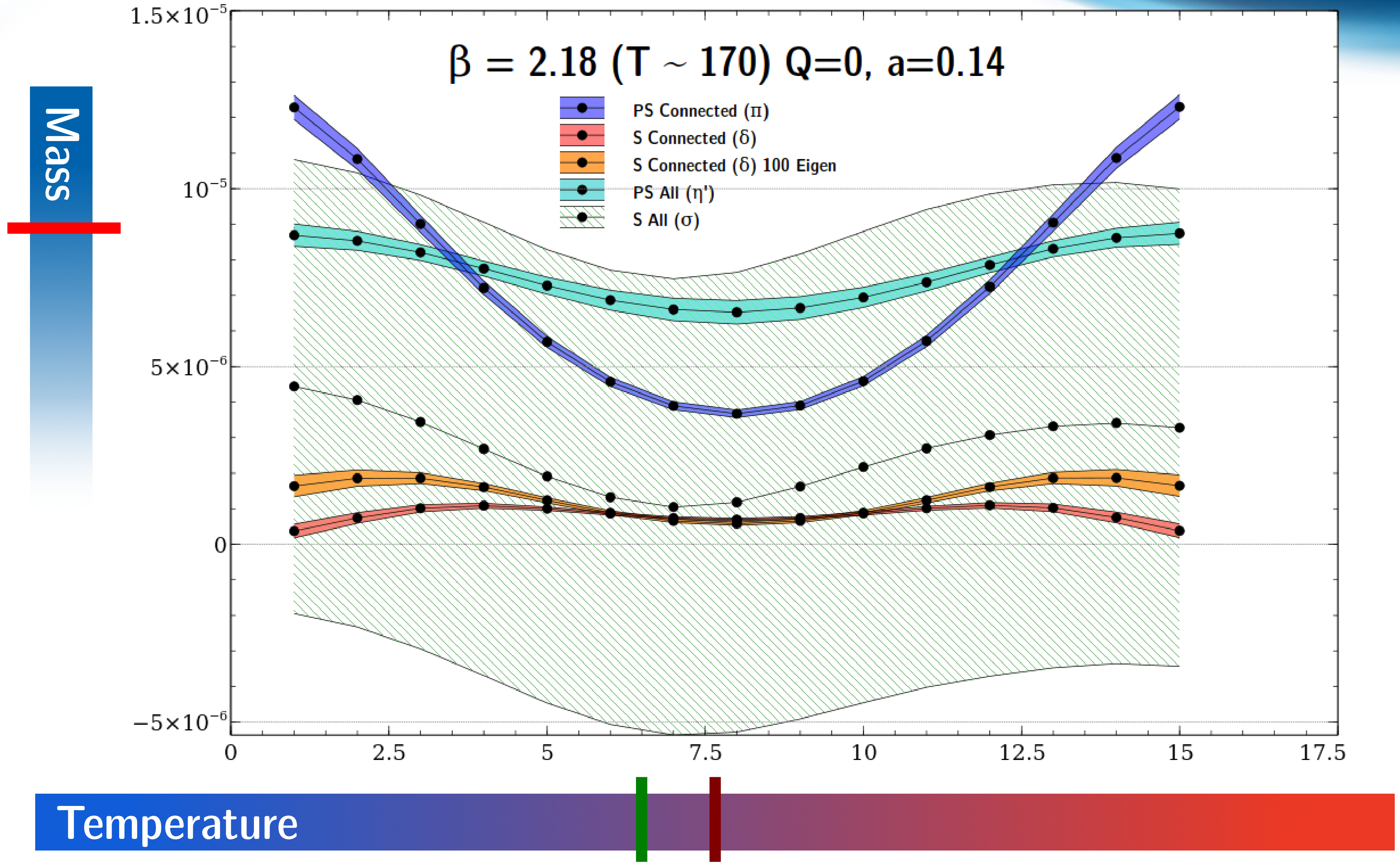
$$\chi^{\pi-\delta} = \int d\lambda \rho_m(\lambda) \frac{4m^2}{\lambda^2 + m^2}$$

If axial symmetry is restored we can conclude that

$$\lim_{\lambda\to 0} \lim_{m\to 0} \frac{\rho_m(\lambda)}{\lambda} = 0$$

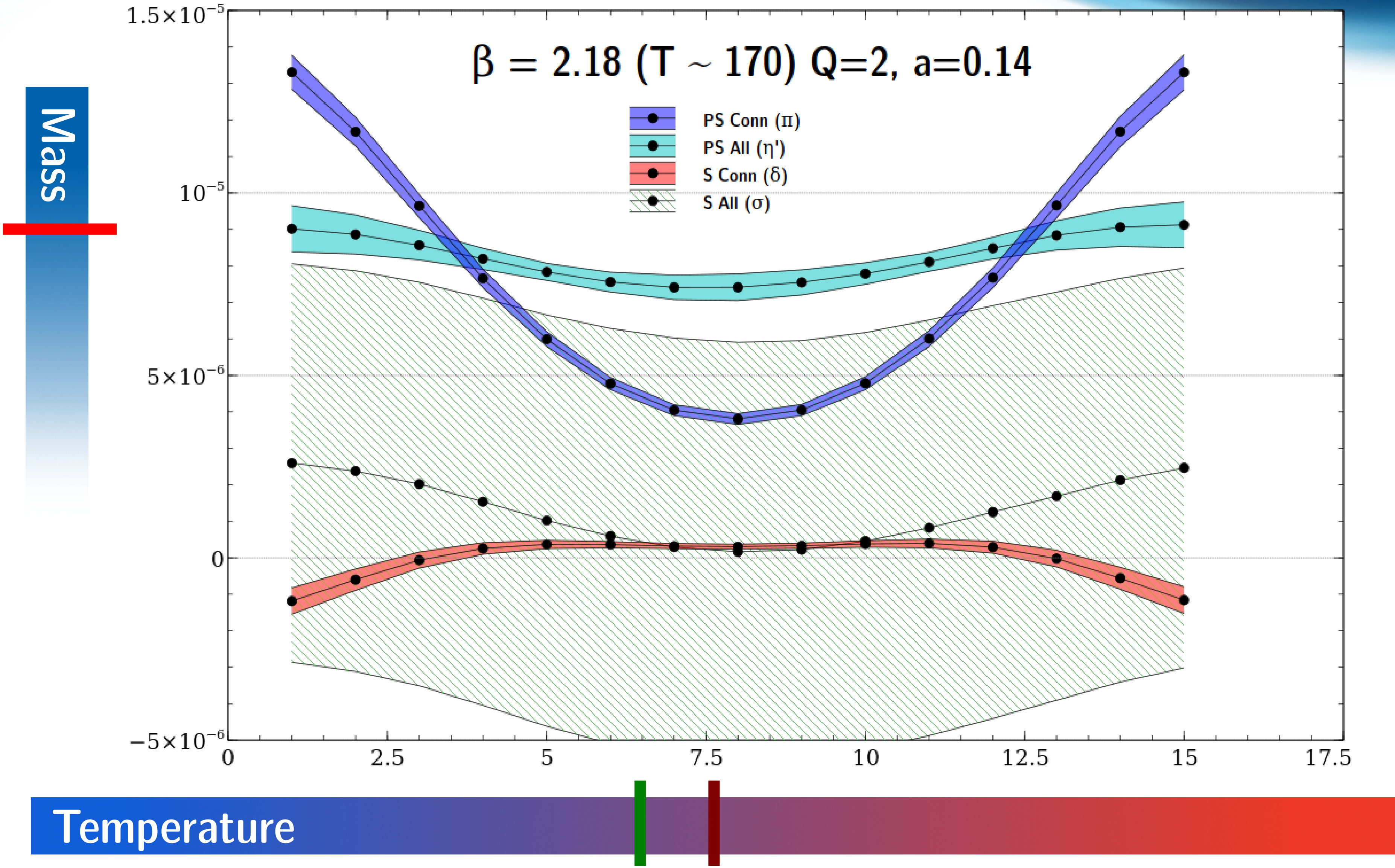
Ref: S. Aoki, internal note

Full QCD – Meson correlators

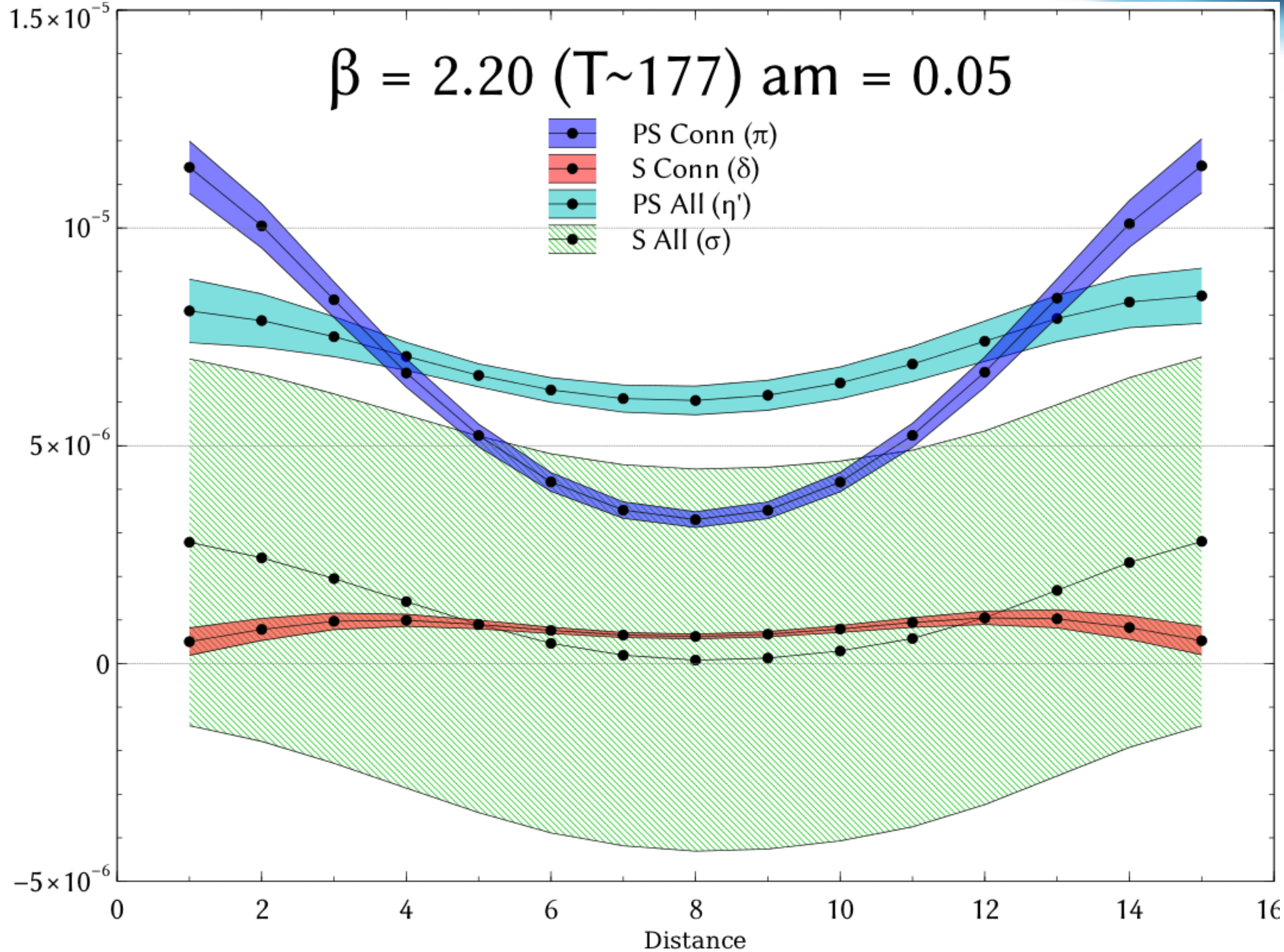


Axial symmetry at finite temp. with overlap fermions

Full QCD – Meson correlators



Full QCD – Meson correlators

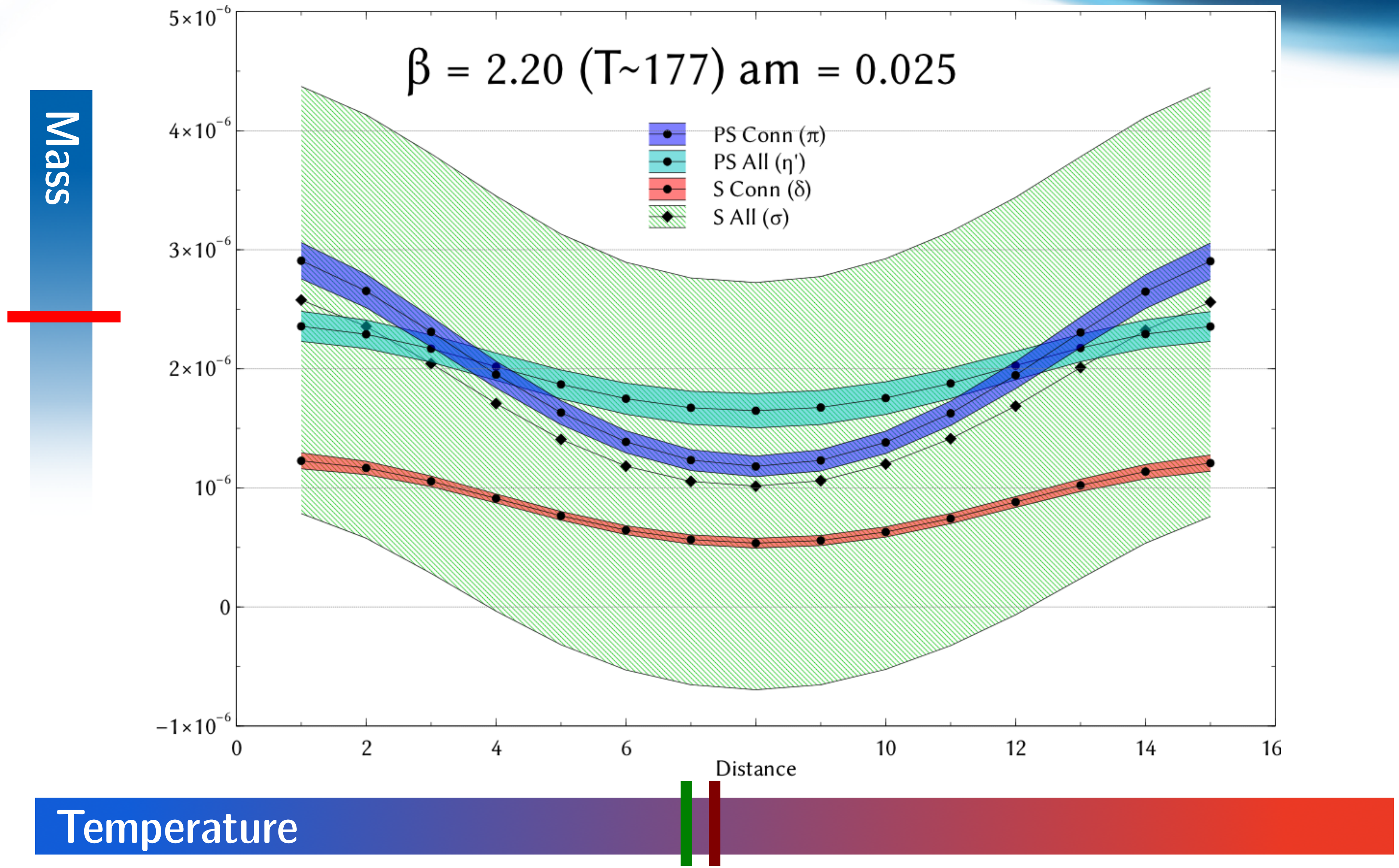


Mass

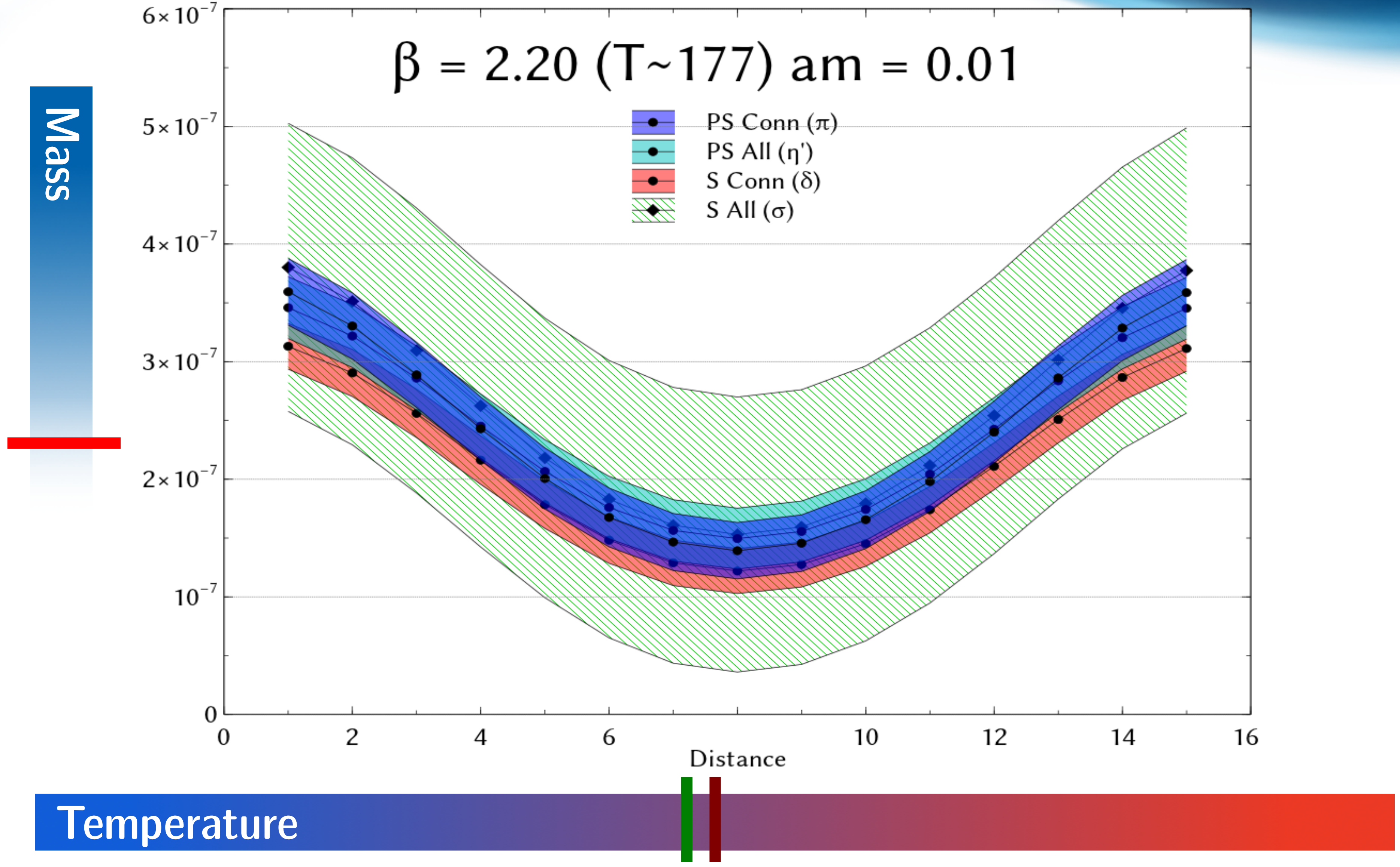
Temperature

Axial symmetry at finite temp. with overlap fermions

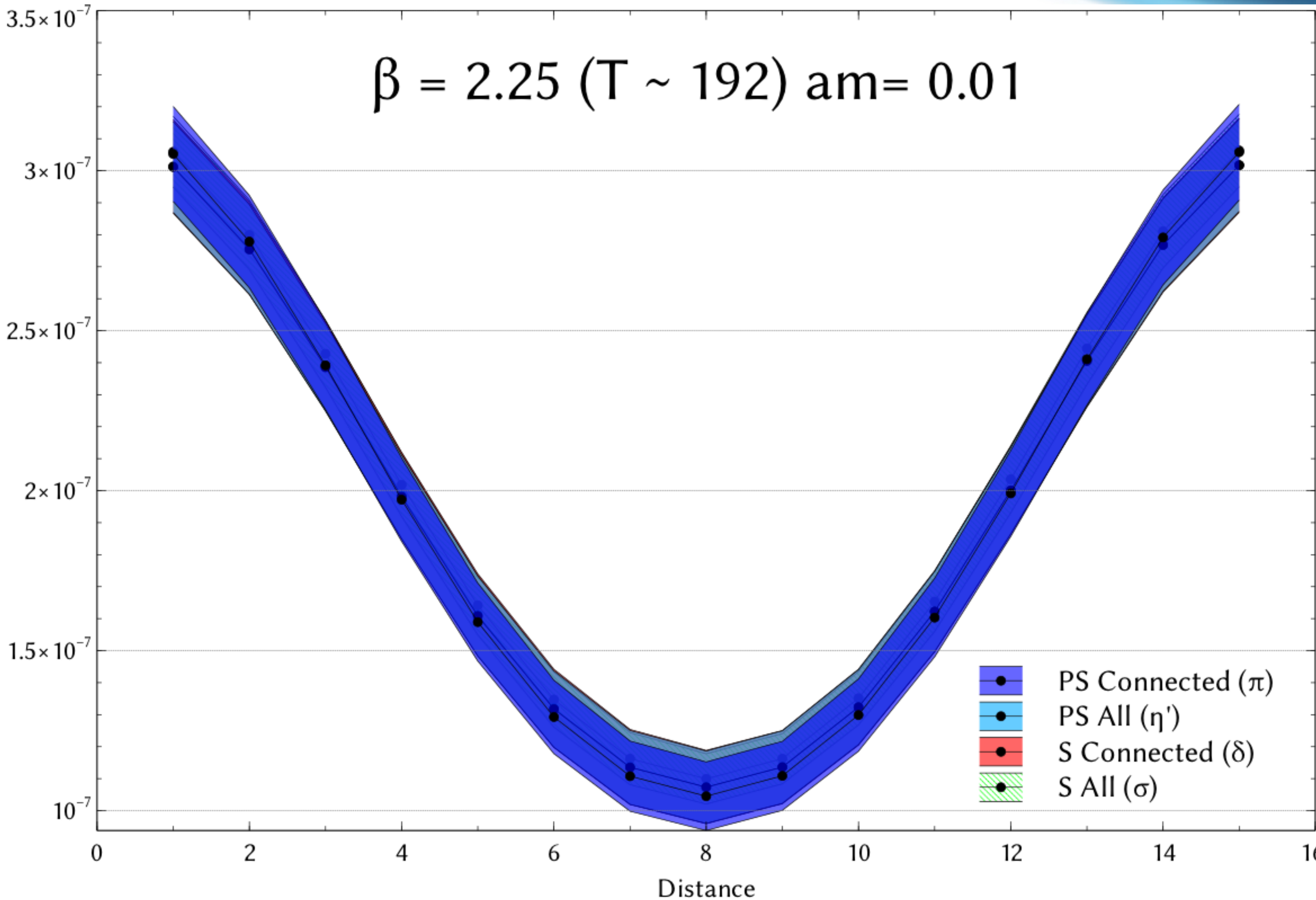
Full QCD – Meson correlators



Full QCD – Meson correlators



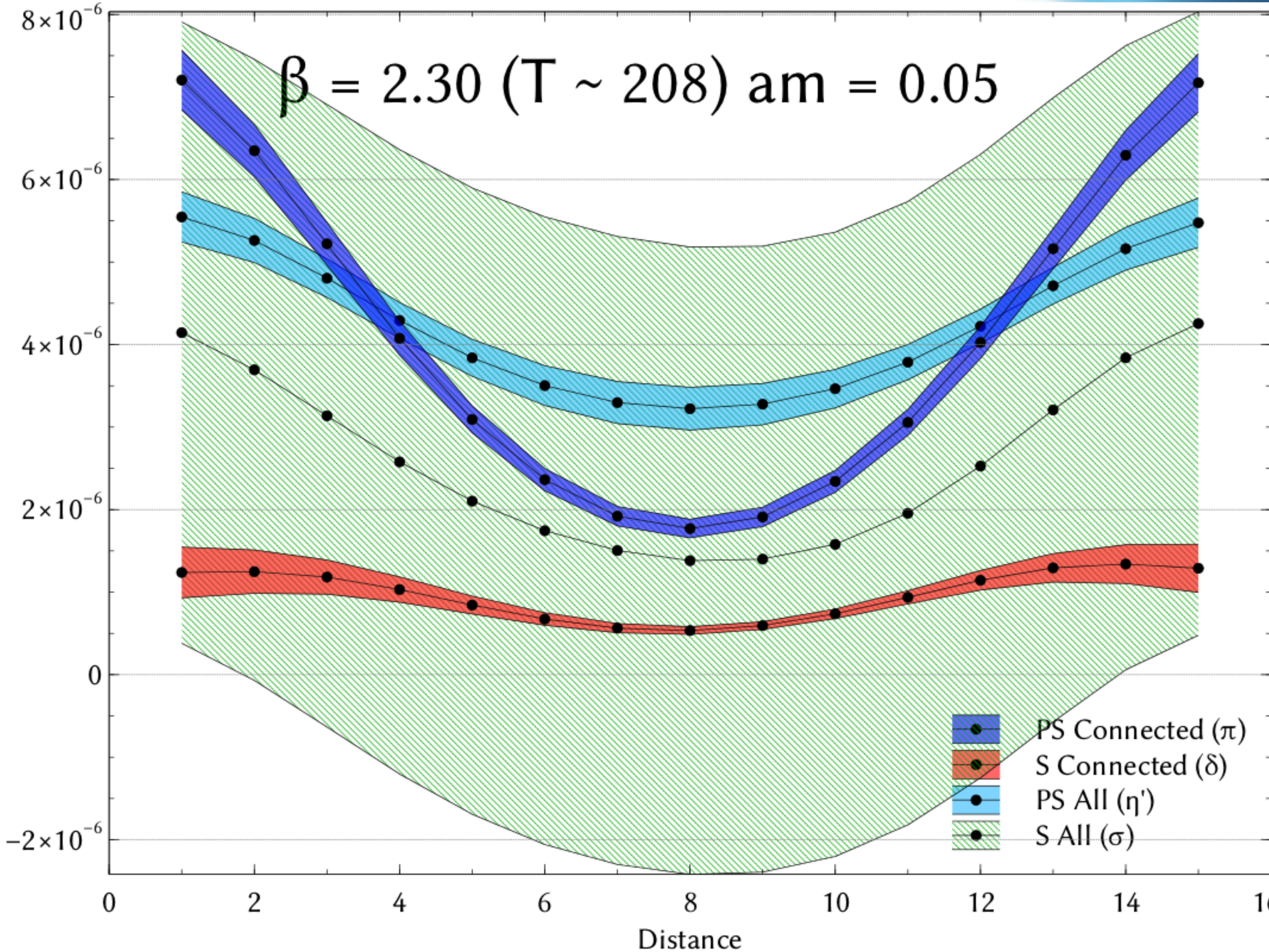
Full QCD – Meson correlators



Mass

Temperature

Full QCD – Meson correlators



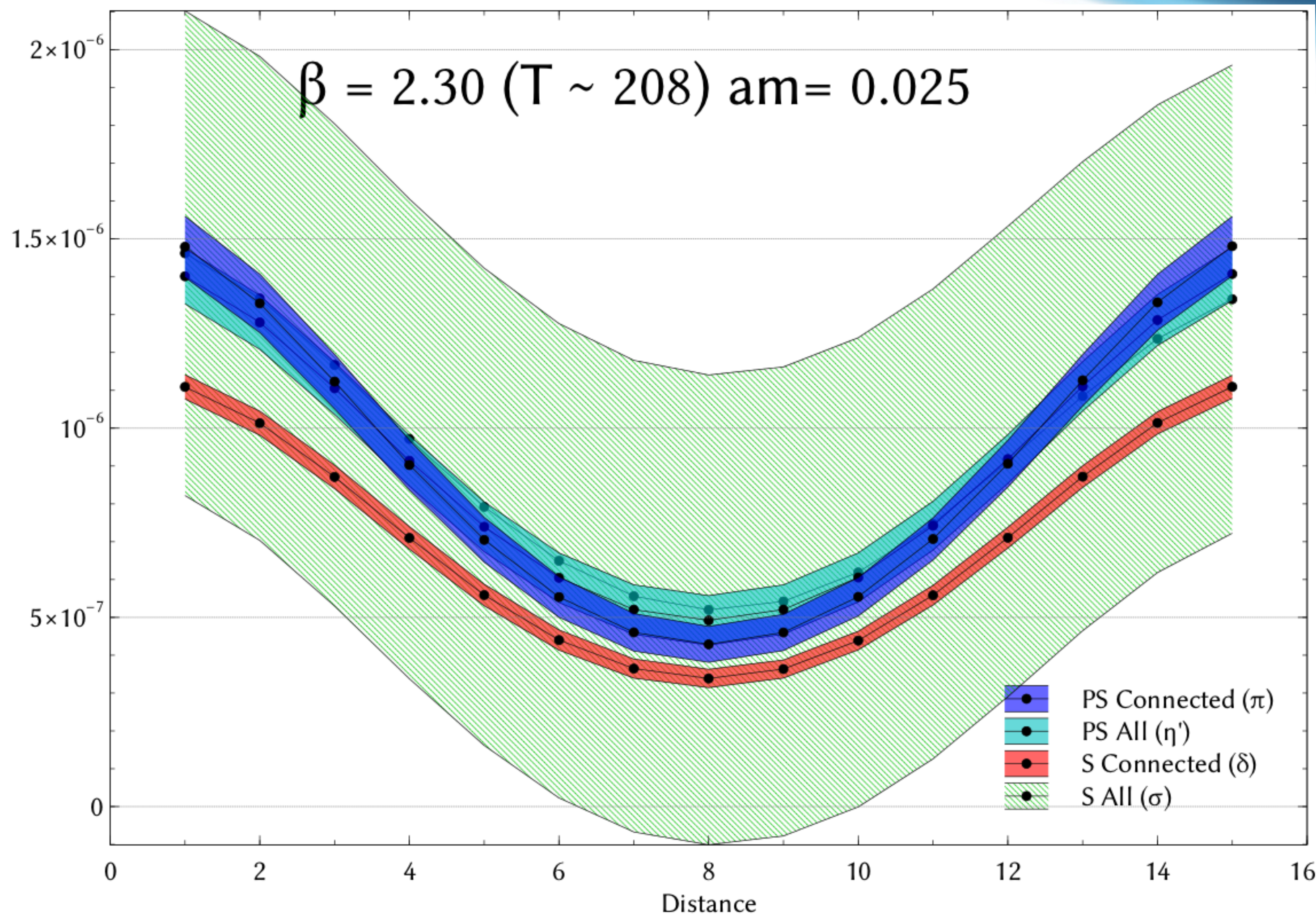
Mass

Temperature

Axial symmetry at finite temp. with overlap fermions

Full QCD – Meson correlators

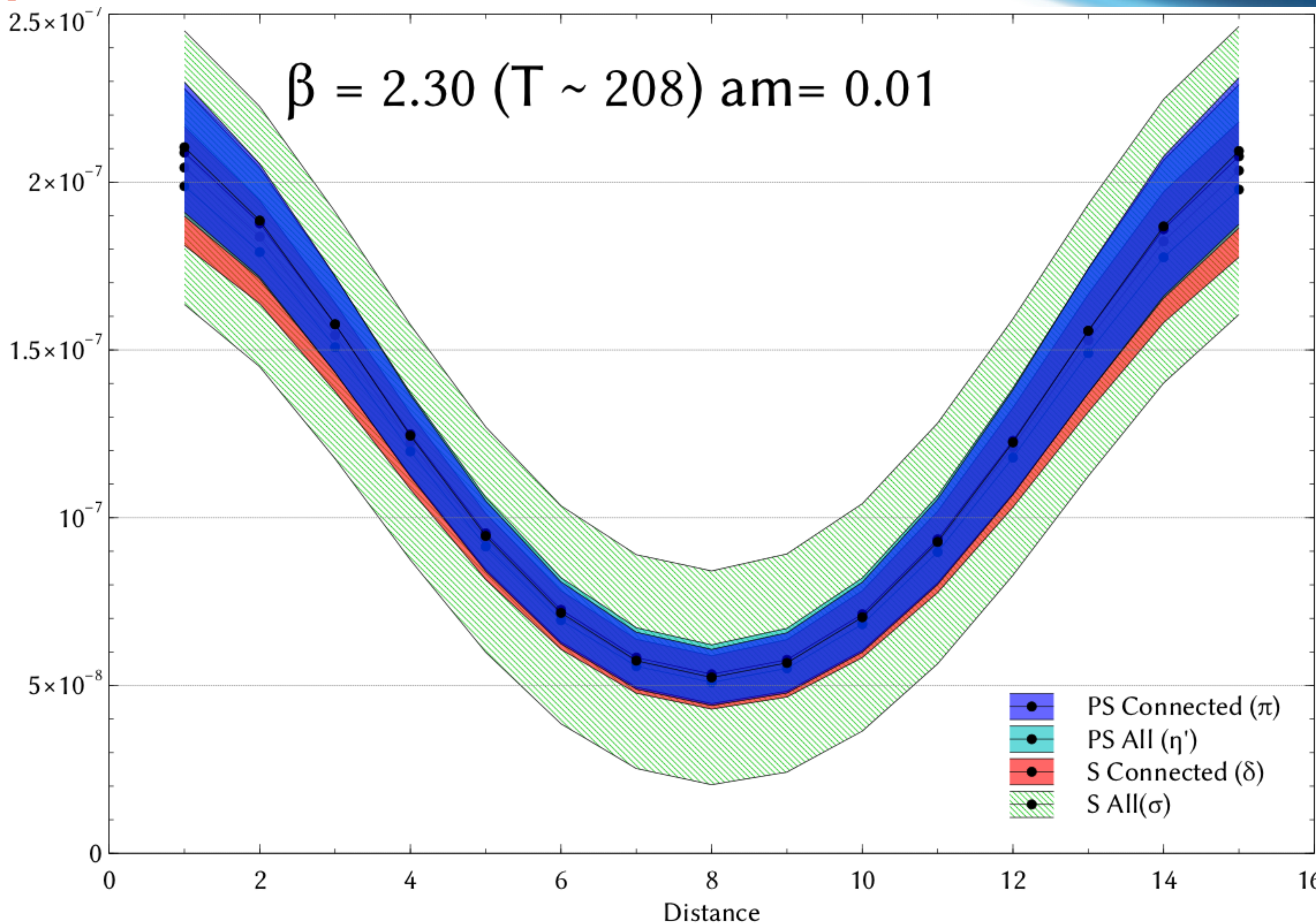
Mass



Temperature

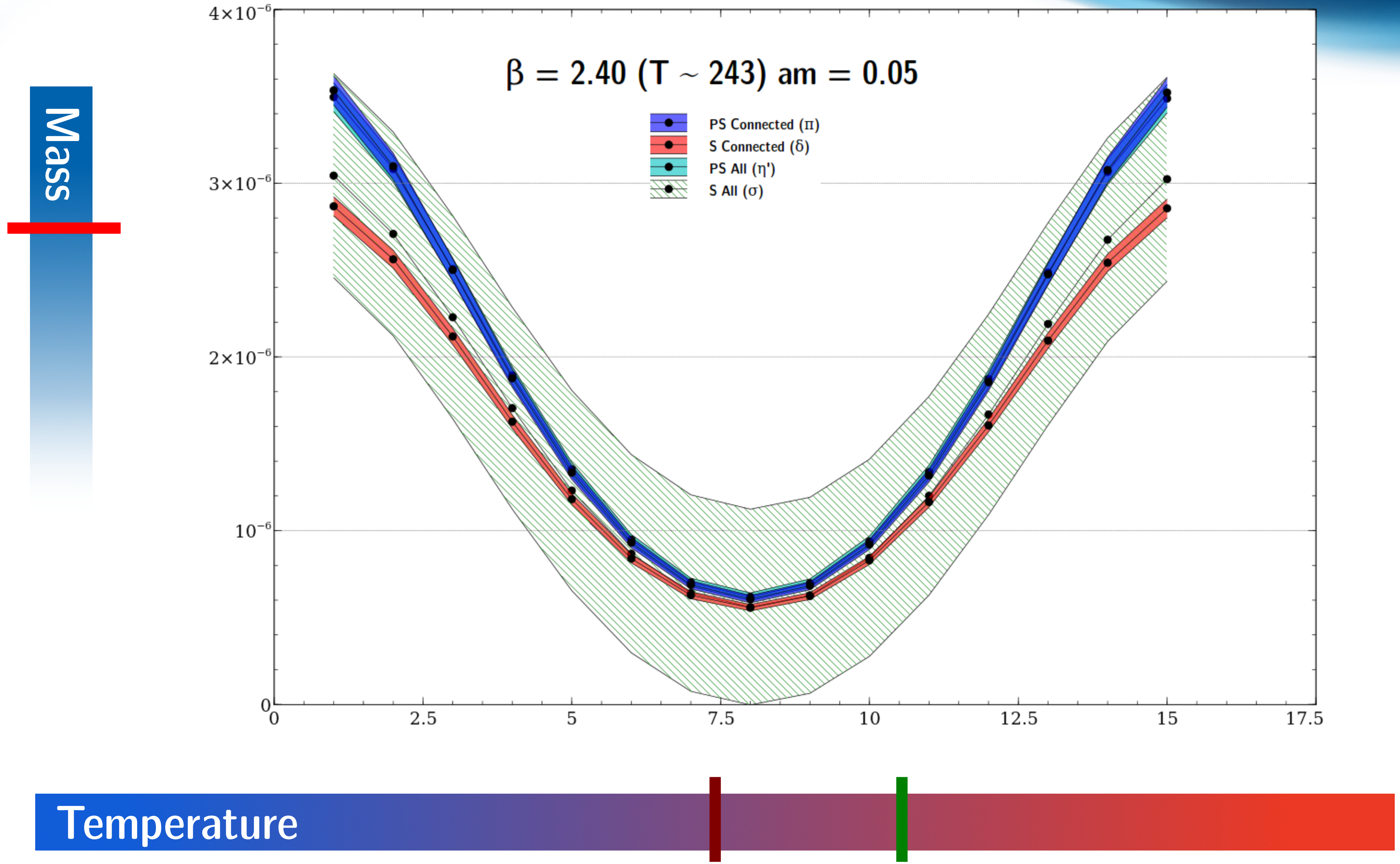
Full QCD – Meson correlators

Mass



Temperature

Full QCD – Meson correlators

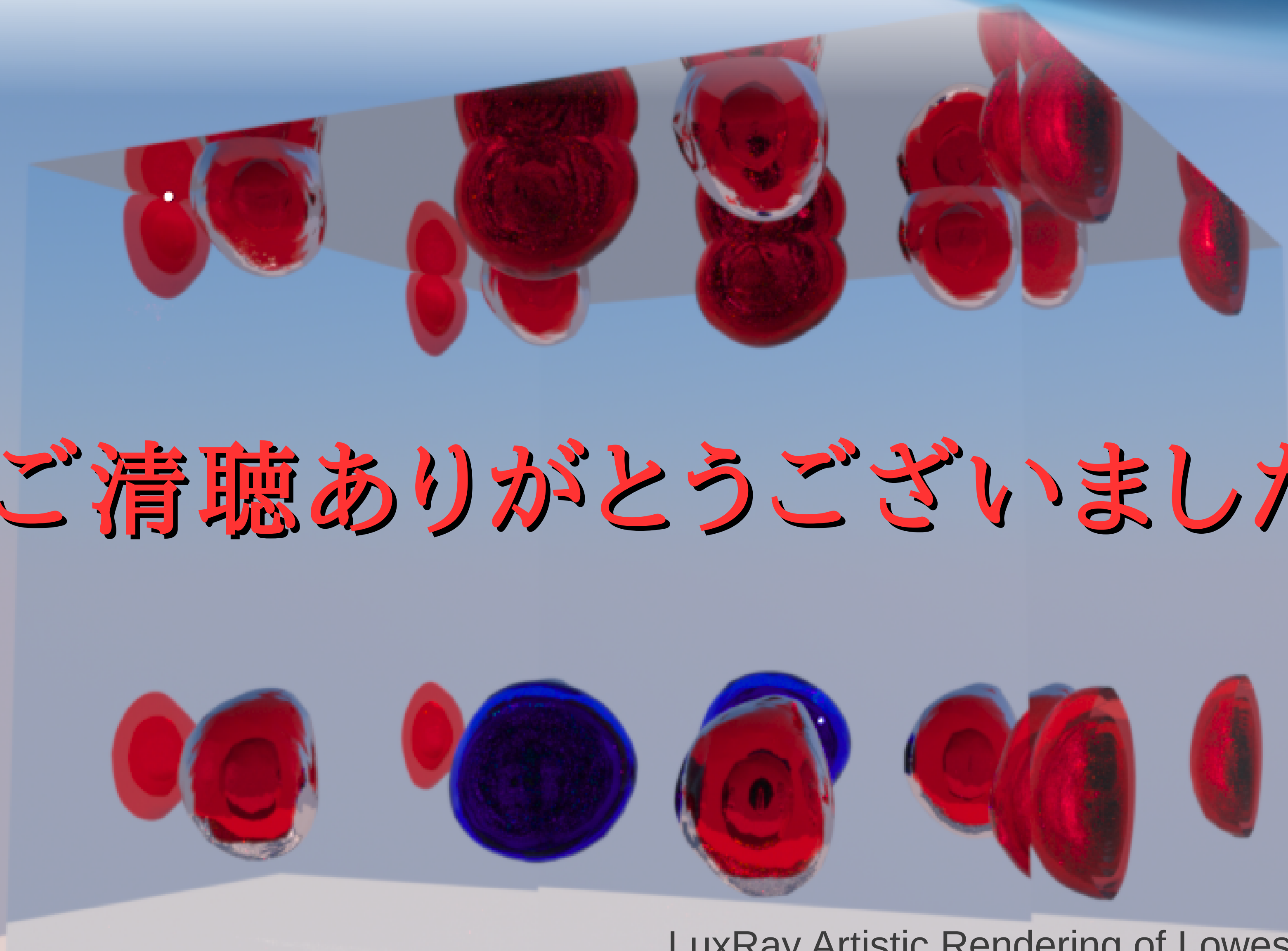


Summary

- Overlap fermions are the best choice to check axial anomaly at finite temperature
- Current machine and algorithms permit now realistic simulations...
- ...at the cost of fixing topology
- We checked feasibility of finite t. sim. by test runs in pure gauge theory
- In pure gauge systematic errors are under control.

- Results in Full QCD show signal of restoration of axial $U_A(1)$ symmetry
- We need 1-2 points more in the chiral limit

ご清聴ありがとうございました



LuxRay Artistic Rendering of Lowest Eigenmode

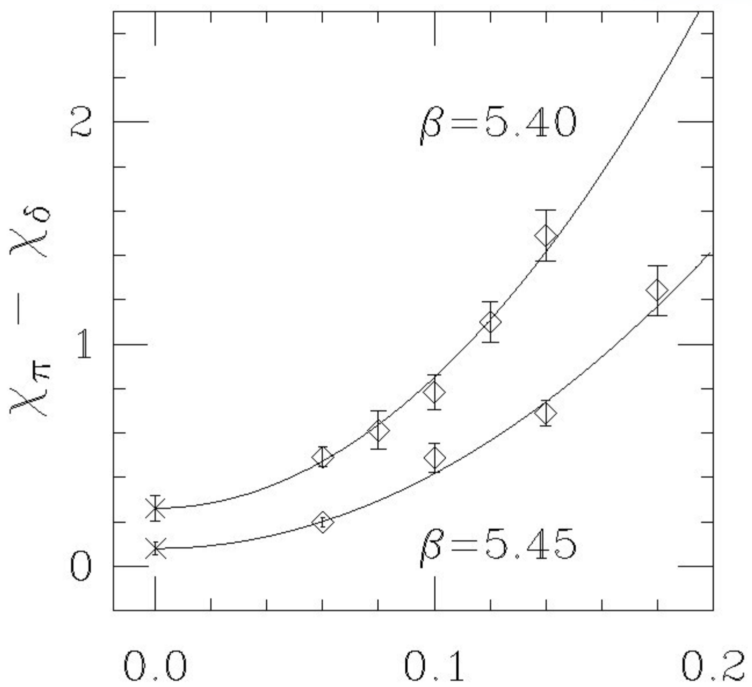
Backup slides

Previous analyses

Vranas (Nucl.Phys.Proc.Suppl. 83 (2000) 414-416)

Measured the difference of susceptibilities χ of π and δ.

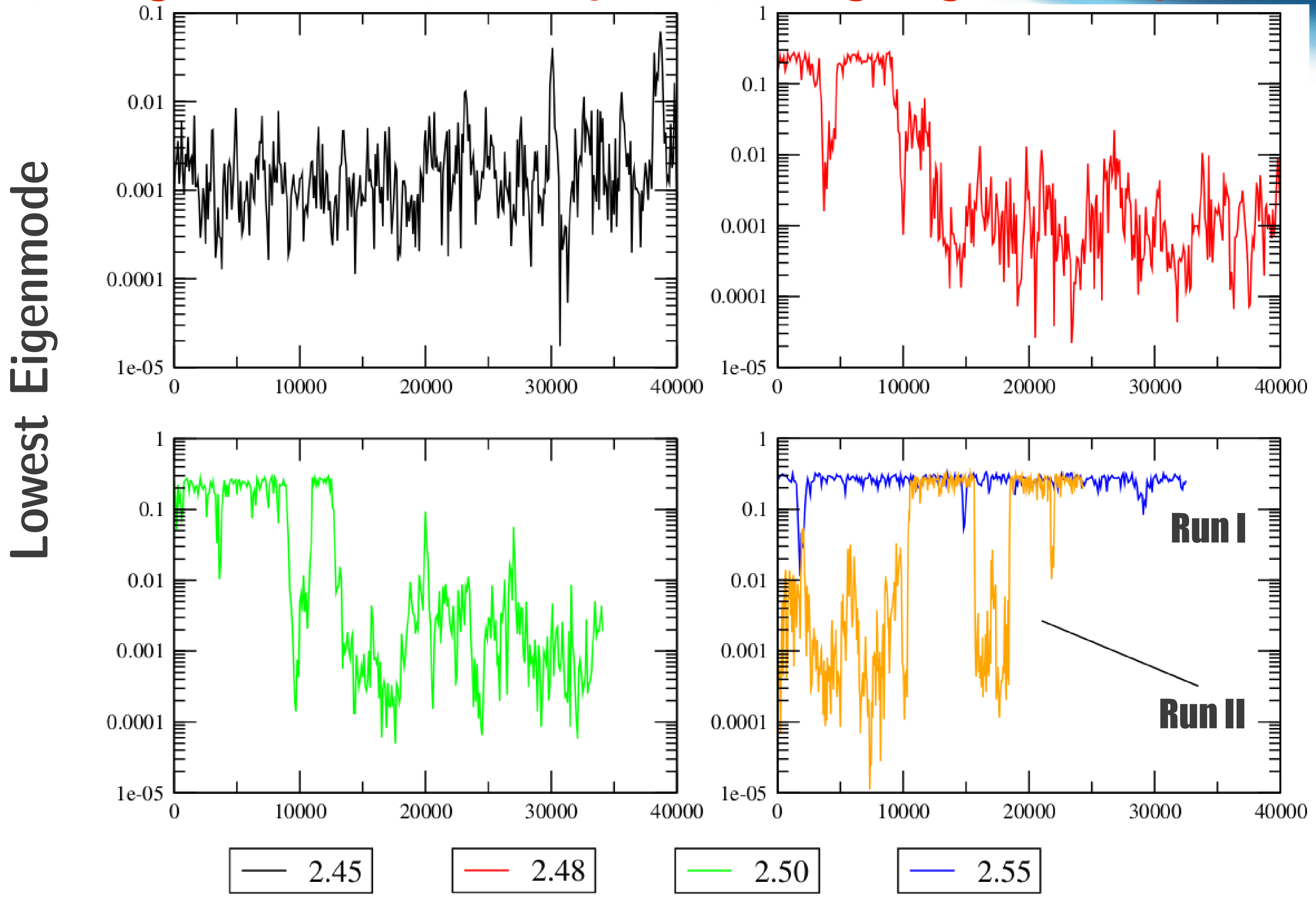
- Action: DWF, $L_s = 24$
- Found a very small relative difference just above T_c in the chiral limit
- Residual mass effects?



Other works with staggered fermions established a restoration at $T > T_c$

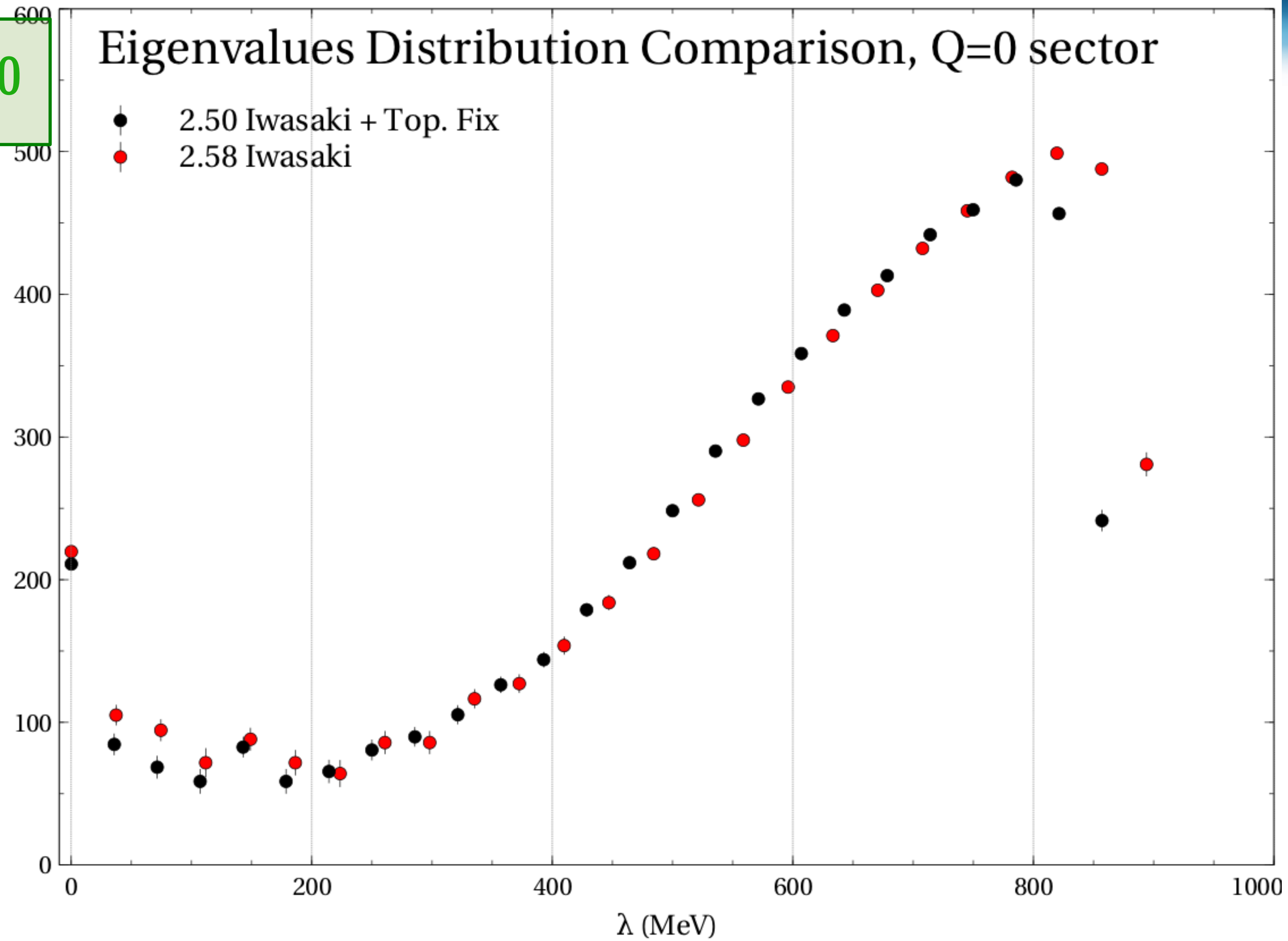
Even bigger problems due to breaking of $U_A(1)$ by staggered fermions.

Topological susceptibility in pure gauge theory - III

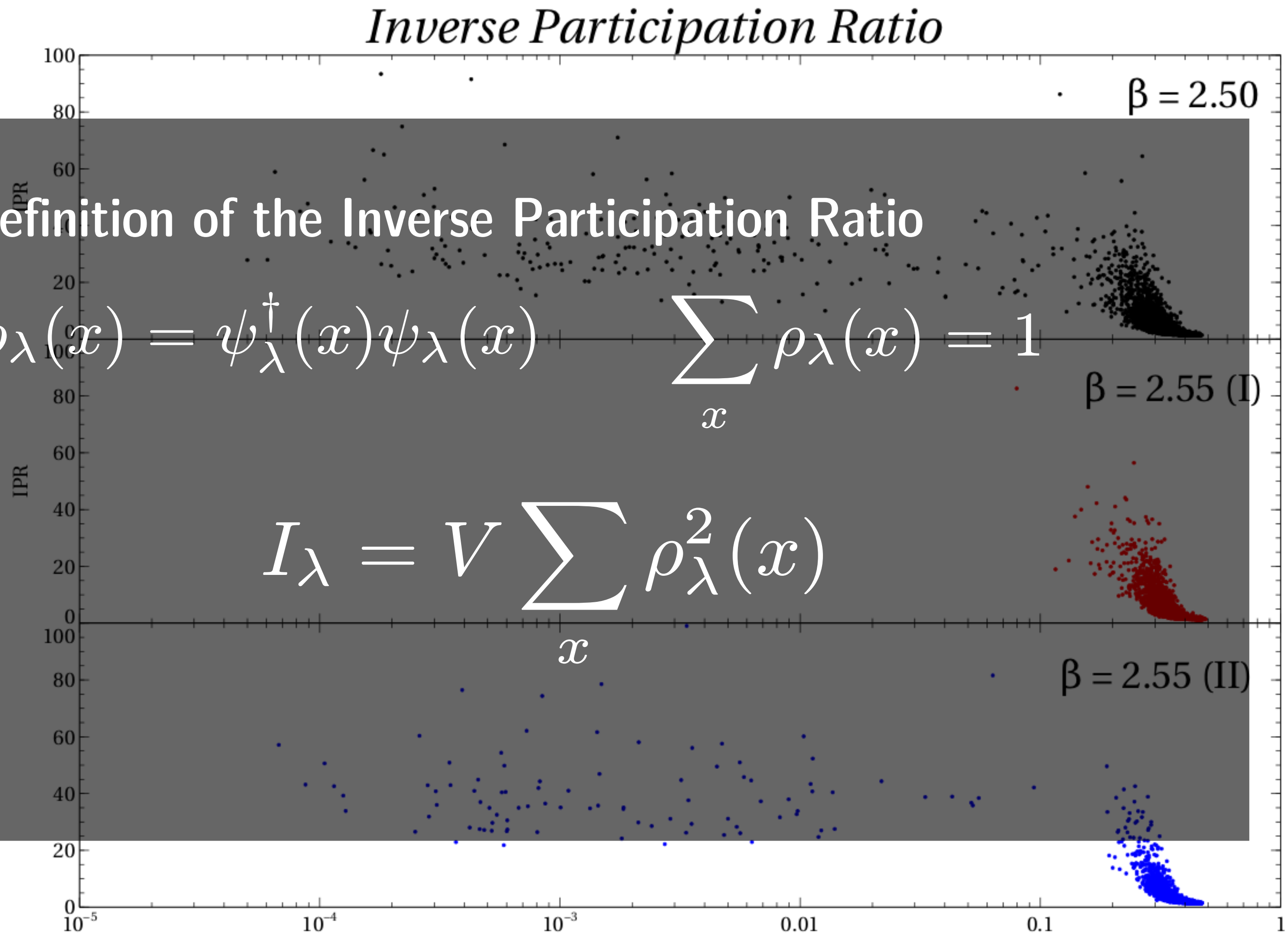


Topological susceptibility in pure gauge theory

$\beta = 2.50$



Topological susceptibility in pure gauge theory - VII

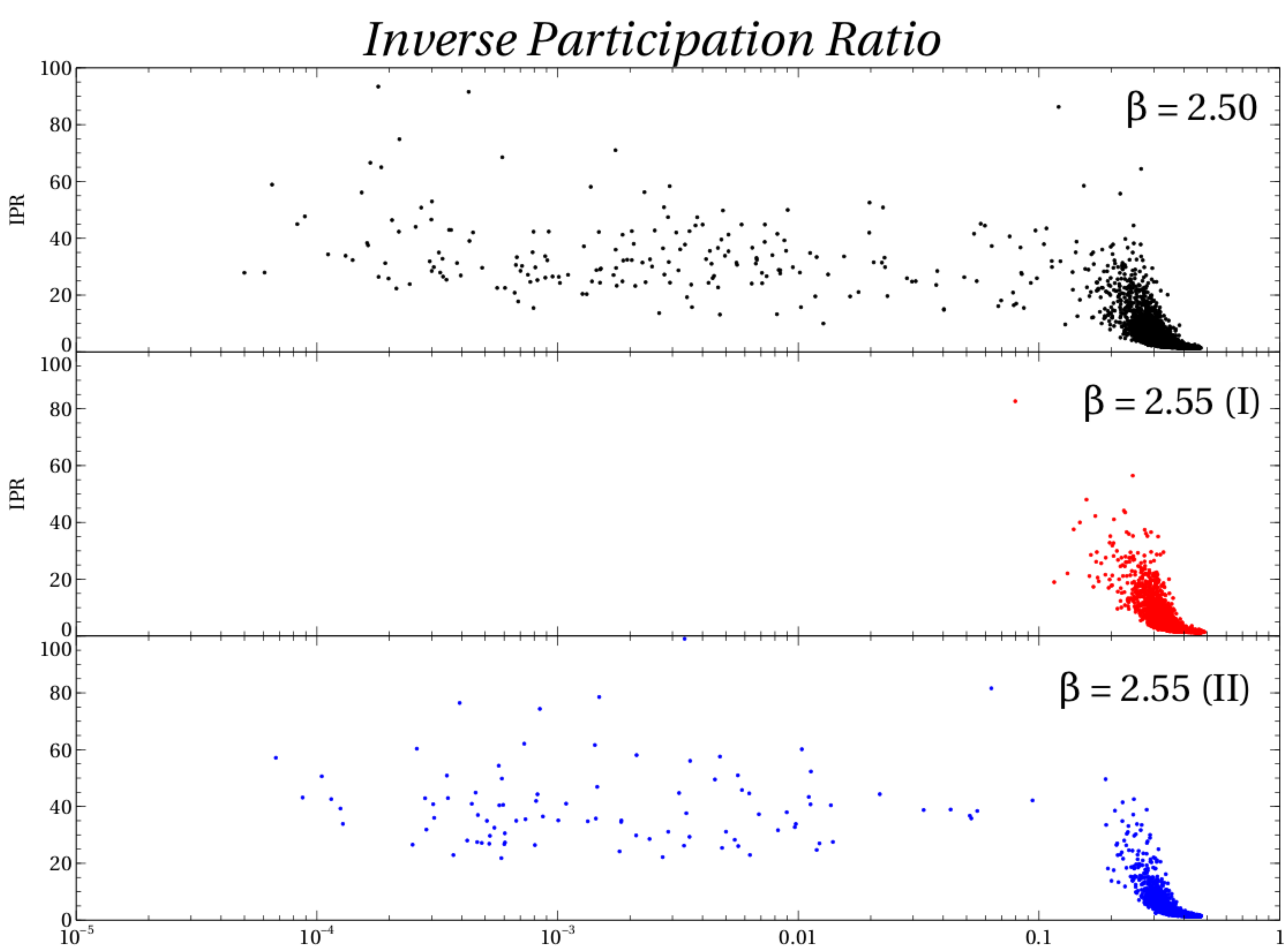


Definition of the Inverse Participation Ratio

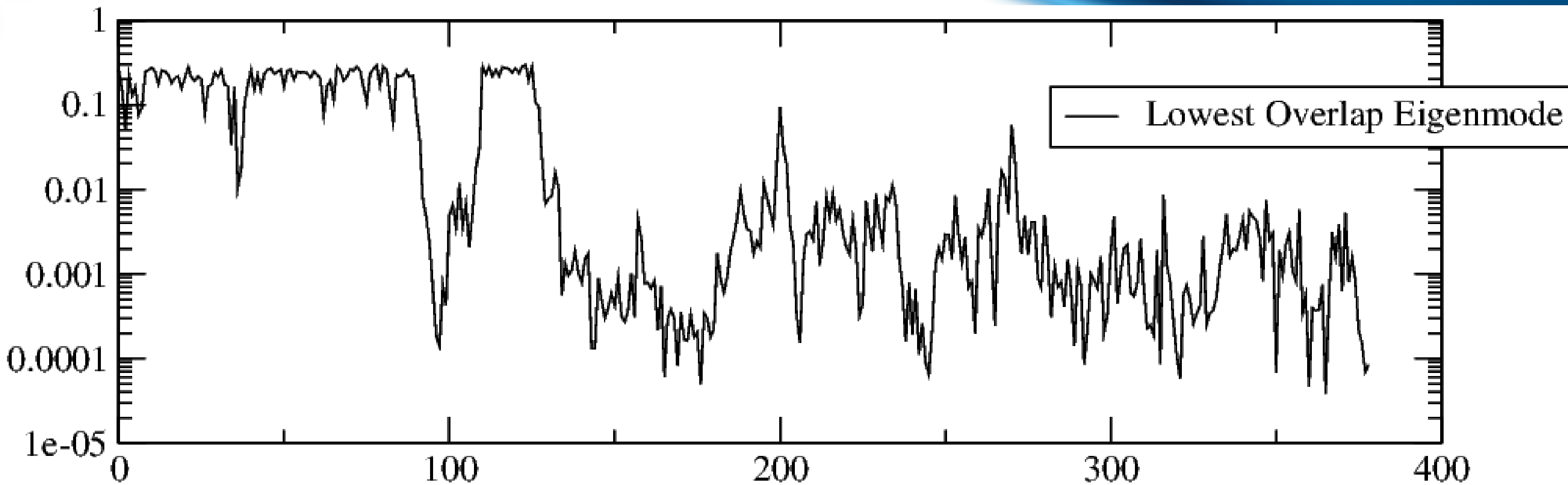
$$\rho_\lambda(x) = \psi_\lambda^\dagger(x)\psi_\lambda(x) \qquad \sum_x \rho_\lambda(x) = 1$$

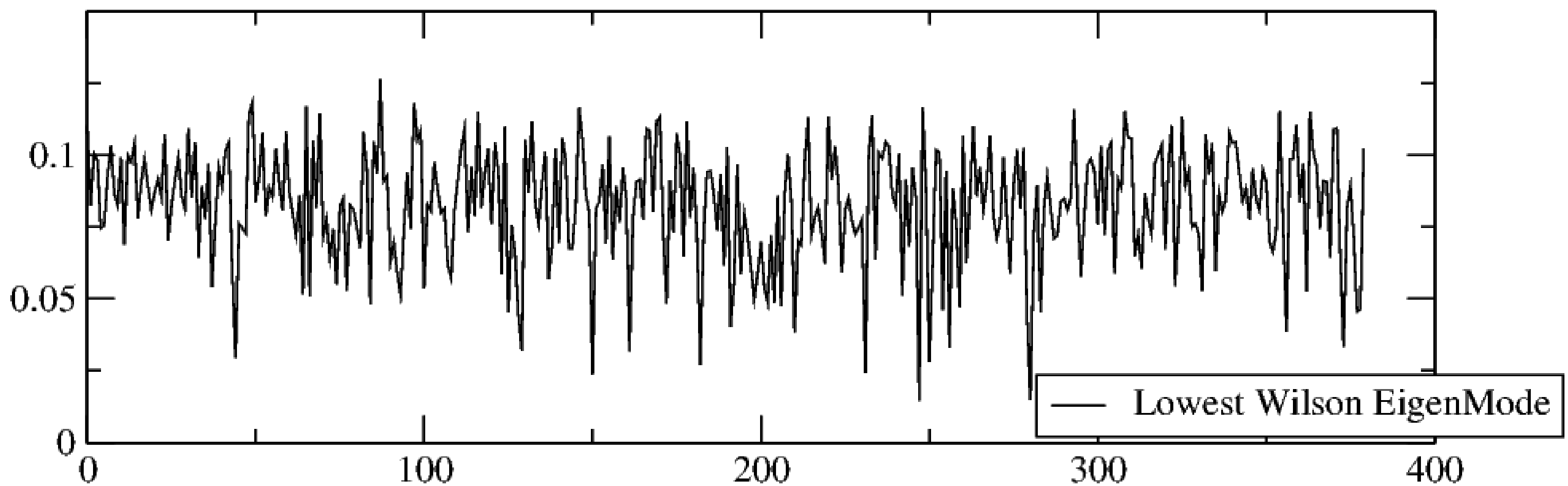
$$I_\lambda = V \sum_x \rho_\lambda^2(x)$$

Topological susceptibility in pure gauge theory - VII

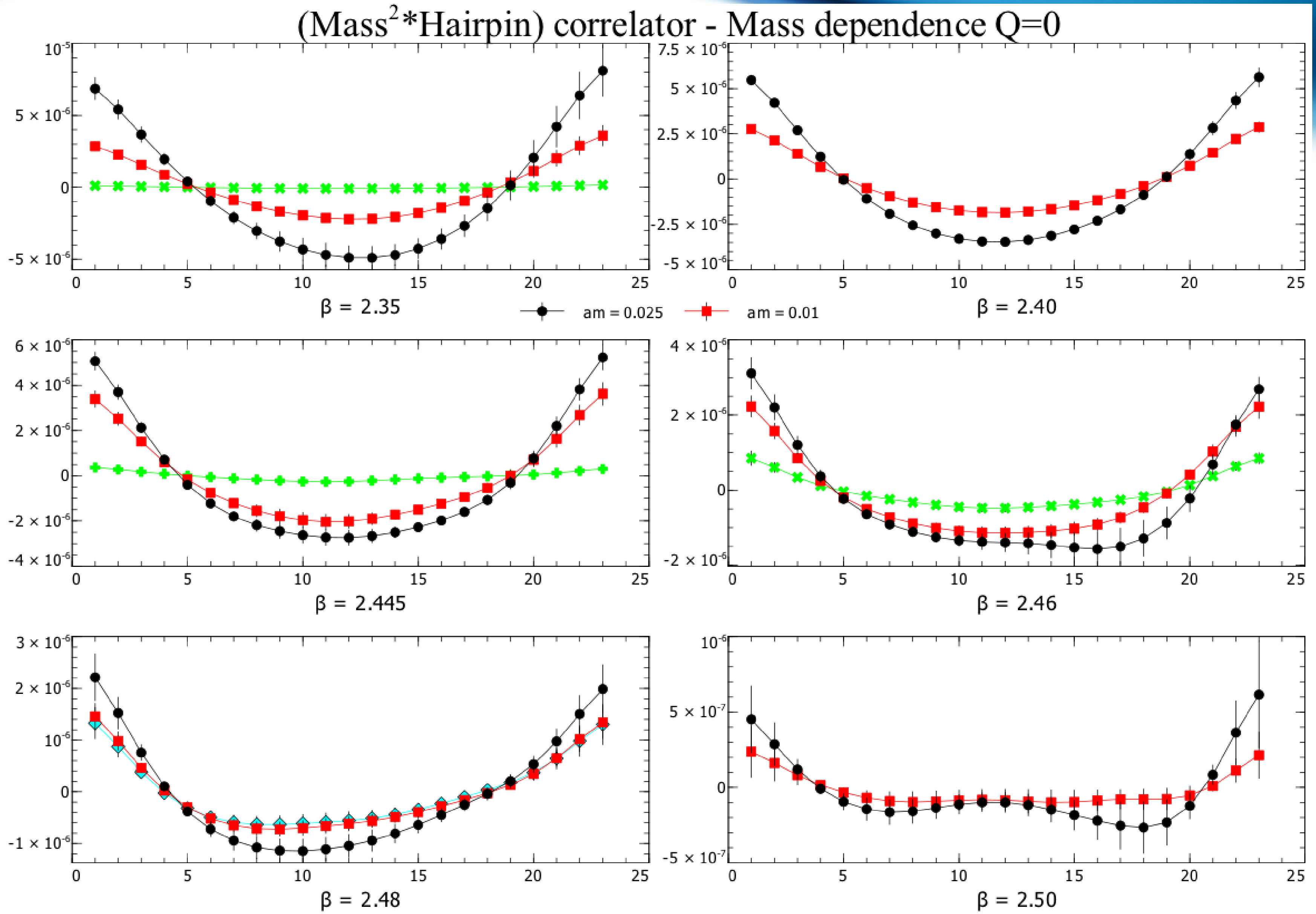


Axial symmetry at finite temp. with overlap fermions

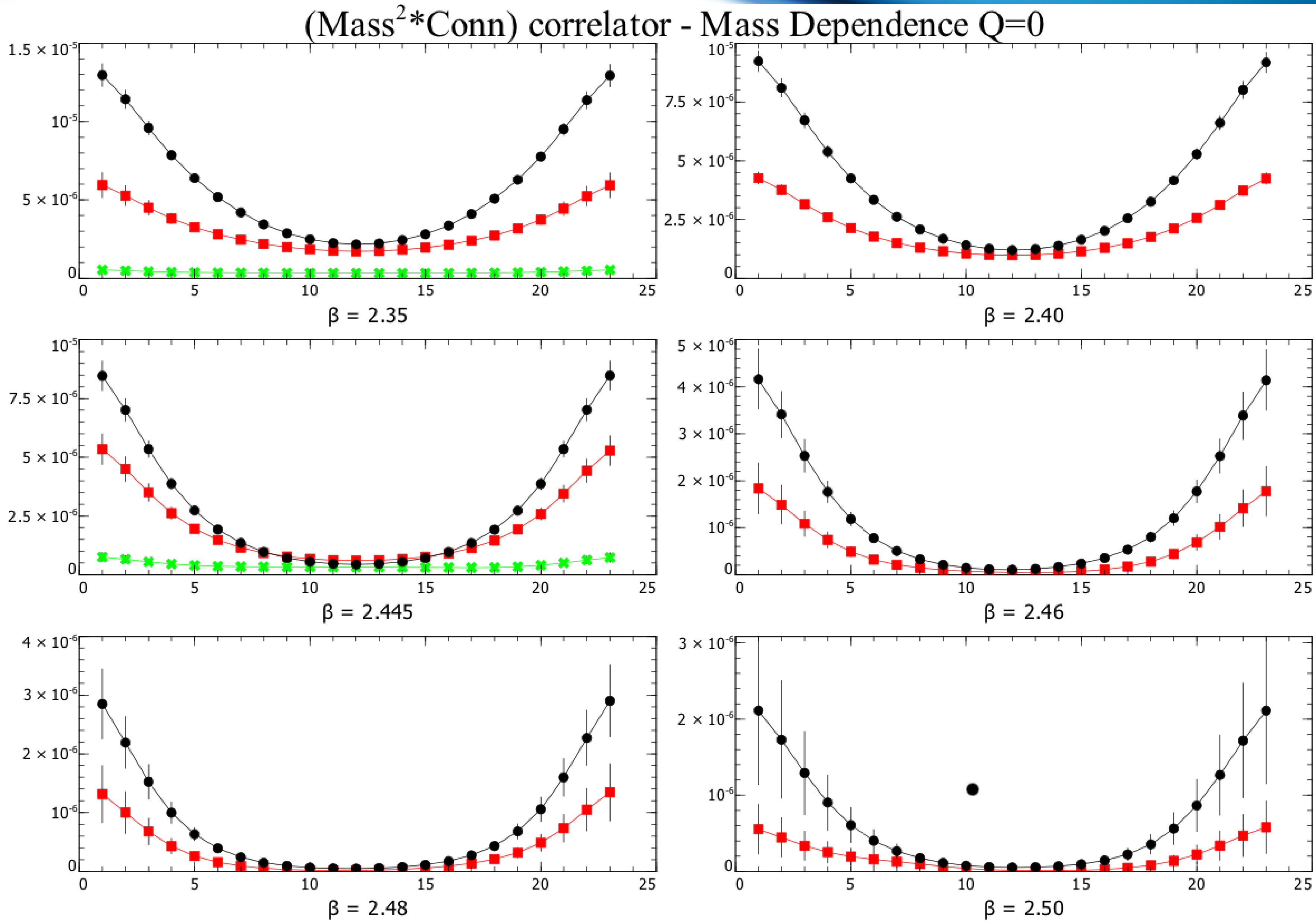


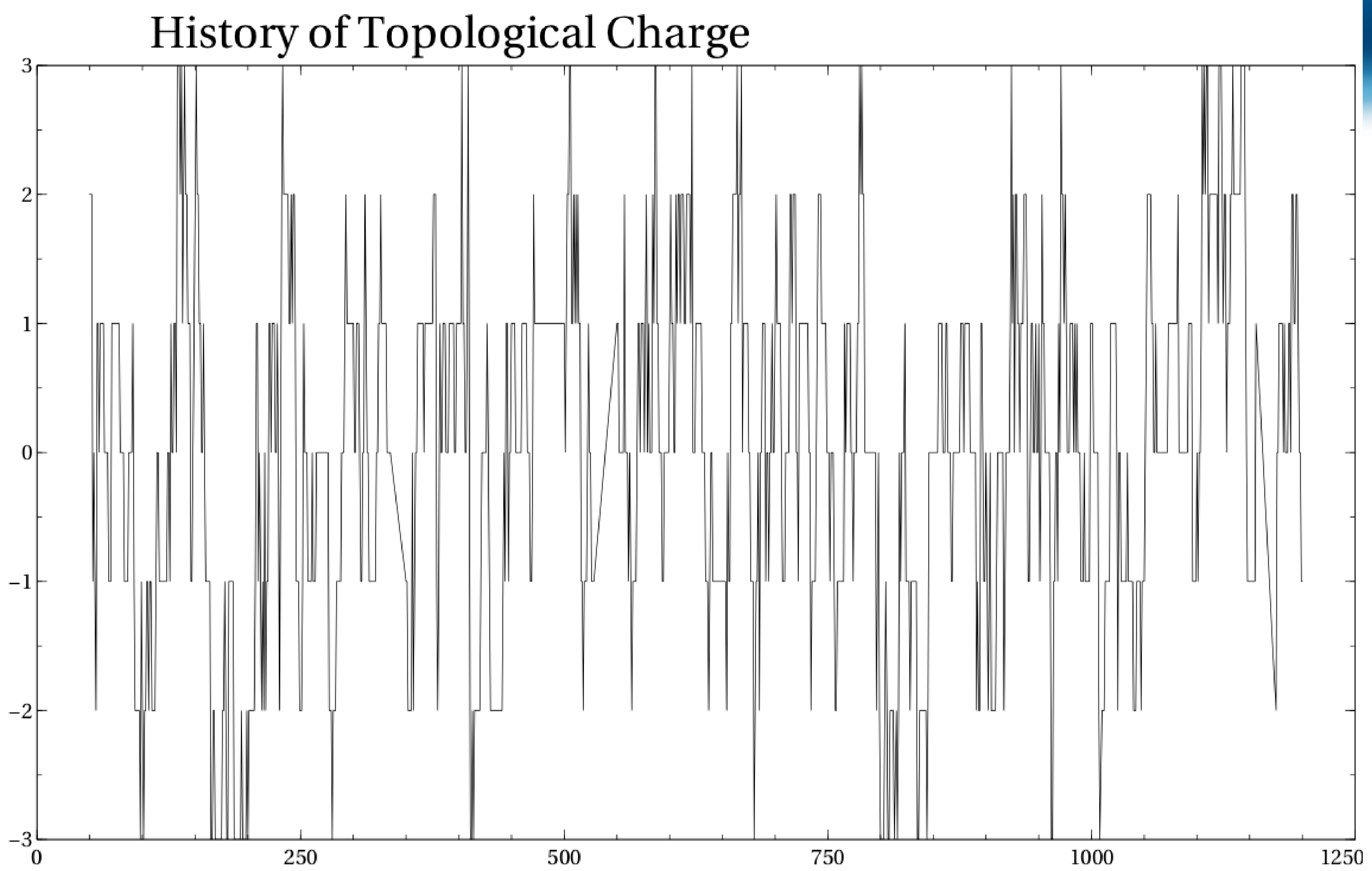


Axial symmetry at finite temp. with overlap fermions



Axial symmetry at finite temp. with overlap fermions




History of Topological Charge
3
2
1
0
−1
−2
−3
0
250
500
750
1000
1250